Statistical Mechanics - Problem set 2

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1. **(Blume-Capel model)** Consider a system of N spin-1 particles placed in a one-dimensional lattice and subject to the Hamiltonian

$$H = -J\sum_{i=1}^{N} S_z^i S_z^{i+1} - D\sum_{i=1}^{N}(S_z^i)^2, \tag{1}$$

where $S_z = \text{diag}(1, 0, -1)$ is the z component of the spin-1 operator for each particle. Discuss the equilibrium properties of this system. Among other things, study the quadrupole moment

$$Q = \frac{1}{N}\Big\langle \sum_{i=1}^{N}(S_z^i)^2 \Big\rangle. \tag{2}$$

2. **(Harmonic dimer)** Consider two quantum harmonic oscillators interacting via the Hamiltonian

$$H = \frac{1}{2}(p_1^2 + \omega_1^2 q_1^2) + \frac{1}{2}(p_2^2 + \omega_2^2 q_2^2) + \frac{g}{2}(q_1 - q_2)^2. \tag{3}$$

Discuss the normal mode decomposition of this model and its equilibrium properties. In particular, play with the fact that $\omega_1 \neq \omega_2$. For instance, check what happens when one frequency is much larger than the other or when both become equal. Despite being apparently simple, there are many fun things one can do with only two coupled harmonic oscillators. For instance, see *Phys. Rev. Lett.* **121**, 040505 (2018).